

Duality invariance of $s \geq \frac{3}{2}$ fermions in AdSS. Deser^a, D. Seminara^b^a*Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, CA 91125 and Physics Department, Brandeis University, Waltham, MA 02454, USA*^b*Dipartimento di Fisica, Università di Firenze and INFN Sezione di Firenze, Via G. Sansone 1, 50019 Sesto Fiorentino, Italy*

Abstract

We show that in $D = 4$ AdS, $s \geq 3/2$ partially massless (PM) fermions retain the duality invariances of their flat space massless counterparts. They have tuned ratios $m^2/M^2 \neq 0$ that turn them into sums of effectively massless unconstrained helicity $\pm(s, \dots, \frac{3}{2})$ excitations, shorn of the lowest (non-dual) helicity $\pm\frac{1}{2}$ -rung and -more generally- of succeeding higher rung as well. Each helicity mode is separately duality invariant, like its flat space counterpart.

Keywords: Electromagnetic Duality, Higher spins

1. Introduction

We address, and complete the answer to, the question whether/how free $m = 0$ spin ≥ 1 systems can retain their known universal ($D = 4$) flat space duality invariance [1] when embedded in (A)dS, rather than flat, backgrounds. Half of the question had actually already been answered in [2], where it was shown that the novel – in dS – PM irreps [3] for $s > 1$ bosons did so (photons always do). The only difference from flat space was, perhaps surprisingly, that rather than having $m = 0$ and just helicities s , they now sported a complete range, $\pm(s, \dots, 1)$ of effectively $m = 0$ helicity excitations, excluding precisely the helicity 0 rung that would have spoiled the duality invariance manifest in each of the higher ones. [Spin 1, being conformally invariant, is a degenerate case, since (A)dS is conformally flat; of course, if studied exactly like its $s > 1$ peers in dS, its duality invariance follows exactly like theirs. Similarly, $s = 3/2$ duality invariance was also exhibited long ago [4] for its “massive”, cosmological SUGRA, version.

For orientation, we recall that the easiest bosonic PM route of [2] uses the dS frame $ds^2 = -dt^2 + e^{2Mt} d\ell^2$, $M^2 \equiv \frac{\Lambda}{3}$; there, one first discovers that, in maximal PM, a particular m/M ratio eliminates helicity-0, leaving a sum of unconstrained helicity $\pm(s, \dots, 1)$ actions. Specifically, for the first non-trivial, $s = 2$ model, the action is that of a transverse-traceless (TT) spatial tensor and a transverse vector (T_i); effective masslessness is achieved by the PM tuning of the two mass parameters (m, M). removing the helicity 0 mode through the residual local scalar gauge invariance of the original action at the PM point, $m^2 = M^2$. However, as we shall see, $m = 0$ models are NOT duality invariant in (A)dS, because their lowest (0 or $1/2$) helicities are reinstated there. It was strongly conjectured that the same process (also explicitly performed for $s = 3$) goes through for ALL s : the auxiliary fields, constraints, etc that necessarily decorate the original covariant actions are gone in the final, non-covariant, unconstrained 3+1 form.

Our spinor models also enjoy PM irreps, but in AdS instead of dS. As mentioned, $s = 3/2$ is the basic, and long known, example of a dual invariant $s = 3/2$ tuned system [4]: In order to obtain the cosmological,

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necessarily AdS extension of SUGRA, one must add a mass term $\sim m\bar{\psi}_m\sigma^{mn}\psi_n$ to its massless action, with the tuning $m \sim \sqrt{-\frac{\Lambda}{3}} = M$. This is exactly equivalent to improving the covariant derivative from D_μ to $D'_\mu = D_\mu + \frac{m}{2}\gamma_\mu$. The effect of this change is to restore the flat space commutativity, $[\partial_\mu, \partial_\nu] = 0 \rightarrow [D'_\mu, D'_\nu] = 0$, thereby restoring the flat space invariance of the model under local spinor transformations, now under $\delta\psi_\mu = D'_\mu\alpha(x)$, and so again removing the lowest, helicity 1/2, excitation [5]. Here, the governing variables are the transverse-traceless and γ_i -traceless spinor-spatial tensors $\psi_{ij\dots}^{tTT\dots}$. The PM invariance [3] always removes the lowest, here helicity 1/2, leaving an effectively massless (upon, legally, field redefining) array of helicities $\pm(s + 1/2, 3/2)$, each separately duality invariant, but now at the above AdS point.

2. Derivation

For compactness, we will freely use the equations and results of [6]; while that work is ostensibly formulated in dS, it is, as noted there, applicable to our AdS format, the change in sign of Λ corresponds to setting the M there to iM ; we will simply keep the dS notation on the above understanding, rather than wasting space with AdS formalism; we also borrow from [2] it the near-certainty that the procedure and results are uniform for all higher spins: again, while higher spin actions require auxiliary fields and constraint variables, these are all absent from the final unconstrained physical 3+1 actions, here for the gamma-and spatial gradient-transverse, traceless spatial tensor-spinor components.

The key, Dirac, equation satisfied by these amplitudes is given by Eq. (14) there:

$$\gamma^0\partial_0\psi^{tTT} + e^{-Mt}\not{\nabla}\psi^{tTT} + [m + (2-s)\gamma^0M]\psi^{tTT} = 0 \quad (1)$$

Clearly, (1) differs from its flat space, massless, counterpart in two basic respects: it contains a “mass” term, $\sim aM\gamma^0 + b m$, as well as the factor e^{-Mt} in the spatial derivative term. The latter is essentially an irrelevant numerical coefficient in the Hamiltonian for spatial duality transformation purposes, also present and harmless for bosons, as explained in [2]. To remove the offending “mass” terms, consider for concreteness $s = 5/2^1$, where we face $-\frac{M}{2}\gamma^0 + m = \frac{M}{2}\gamma^0 + (m - \gamma^0M)$. The parenthesis vanishes at the PM point, because γ^0 is diagonal with $\pm i$ entries, provided we add, beyond $m^2 + M^2 = 0$, the fermionic requirement that the upper/lower components of ψ obey its respective roots $\pm im + M = 0$. The remaining, $M/2\gamma^0$, term is simply removed by rescaling ψ by $\exp(\frac{M}{2}t)$, to leave the sum of flat space pure helicity $> 1/2$ actions (modulo the irrelevant e^{-Mt} term in the Hamiltonian). While one might worry that any amount of M -dependence can be removed this way, the process here is really an artifact of the AdS gauge choice: had we proceeded in conformal AdS gauge,

$$ds^2 = (MT)^{-2}(-dT^2 + d\ell^2), \quad (2)$$

from the start, we would have found the fully flat form of (1),

$$\left[\gamma^0\frac{\partial}{\partial T} + \not{\nabla}\right]\psi^{tTT} = 0, \quad (3)$$

since PM actions are all conformally invariant (indeed, that is their special virtue). We can also recover (3) from the, final, massless (1),

$$[\exp(Mt)\gamma^0\partial_0 + \not{\nabla}]\psi^{tTT} = 0, \quad (4)$$

by performing the (trivial) gauge transformation from our t - to the T -frame (2). But (4) is just the flat space, E-B, form given in [4] namely

$$\gamma^0E + B = 0. \quad (5)$$

This is both manifestly ($E \leftrightarrow B$) rotation invariant, and a time-local canonical transformation, in terms of the underlying canonical pair, as detailed in [1] for all spins. Indeed, the same, natural, conformal frame

¹Different values of the s can be reduced to this case by performing the field the redefinition $\psi^{tTT} \mapsto e^{(s-5/2)Mt}\psi^{tTT}$.

could have been used for the bosonic case [2] directly, or also reached by transforming to T-frame there, to remove the $\exp(MT)$ factor in the corresponding t-frame Hamiltonian there, starting from its PM form,

$$\mathcal{L}_{\text{boson}} = p^a \dot{q}_a - \exp(-MT) \frac{1}{2} [p^2 + q^2] \quad (6)$$

where the summed index a runs over all helicities > 0 (or > 1 etc., in the various other PM levels discussed below).

We remark finally that for spins $> 3/2$, there exist different PM levels, each excising more lower helicities, until only helicity $\pm s$ is left. Each of these occurs at different m/M ratios, and each is duality invariant by tuning M to remove the m term in the corresponding Dirac equation, then rescaling the spinor-tensor amplitude to remove whatever M -dependence remains. In this sense there is in fact a much larger set of dual-invariant PM-levels for any s , than the unique $m = 0$ one in flat space.

3. Summary

We have shown that all $s \geq 3/2$ PM free fermionic models in suitably PM tuned AdS are duality-invariant under the same transformations as in flat space, separately for each effectively massless helicity ($> 1/2$) component. Together with the existing—essentially identical bosonic PM duality invariances in dS [2], this establishes the maximal curved spacetime generalization of flat space free higher spin field duality invariances.

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